A model for bubble formation and weeping at a submerged orifice with liquid cross-flow

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Abstract

A new theoretical model to predict bubble frequencies and weeping rates at a submerged orifice with liquid cross-flow has been developed. The model predicts a significant influence of liquid cross-flow velocity on bubble formation frequency, and especially on liquid weeping. Simulated values of weeping rates for different orifice diameters, gas flow rates and liquid cross-flow velocities show good agreement with experimental data.

Keywords: Bubble formation; Weeping; Liquid cross-flow; Mathematical model; Orifice; Sieve trays

1. Introduction

Liquid weeping is a significant phenomenon which may adversely affect the performance of sieve tray processes such as distillation, absorption or bubble columns. The coupled mechanism of bubble formation and weeping at an orifice in a quiescent liquid has been successfully modeled (Zhang & Tan, 2000). However, in many bubbling situations, bulk liquid flow is a distinct feature of the process, e.g., liquid circulation in bubble columns or liquid cross-flow across a sieve tray in distillation/absorption processes. The role of liquid cross-flow in influencing bubble formation is fairly understood (Tsuge, Hibino, & Nojima, 1981; Marshall, Chudacek, & Bagster, 1993; Tan, Chen & Tan 2000) and consensus has been reached that liquid cross-flow velocity significantly increases bubble frequency, and thereby decreases bubble volume. However, the role of cross-flow on weeping has not been systematically studied, although some workers have suggested the possible liquid cross-flow effects on reducing weeping rates.

Zanelli (1975) described a modified weir design for sieve trays where a portion of the liquid was allowed to flow through a horizontal slot part way up the weir. This gave the weir some of the characteristics of an orifice with transverse liquid flow, and weeping was reduced using the device. Ponter and Tsay (1983) designed an apparatus to simulate the operation of a sieve plate in a distillation column, and investigated the bubble contact angle at and above the incipient weeping regime under liquid cross-flow. The authors claimed that a minimum contact angle was exhibited at the incipient weeping point with a corresponding maximum residence time. However, no further quantitative work was done in terms of liquid cross-flow velocity and weeping rates. Lockett and Banik (1986) studied weeping phenomena in detail by using a simulated plate column. The authors observed some rather unusual behavior when using a combination of low weir height (25 mm) and high liquid load (60 ~ 90 m^3 m^-1 h^-1), i.e., weeping tended to be lower than expected under these circumstances. The possible reason was attributed to the high horizontal velocity of the liquid crossing the tray. Marshall (1990) reported a phenomenon in their study of bubble formation at a submerged orifice with liquid cross-flow, i.e., when the liquid cross-flow velocity was above a critical value, intermittent bubble formation turned into continuous and no weeping was observed even for low gas flow rates with large orifices.

This paper presents a new model for bubble formation and liquid weeping at a submerged orifice with liquid cross-flow. Potential flow theory is used to represent liquid pressure around the bubble. Weeping may occur during the period between successive bubbles and is influenced by the instantaneous difference between liquid pressure and gas chamber

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pressure. Experimental results for measured values of bubble frequency and weeping rates are compared with model predictions.

2. Model development

The present model is an extension of an earlier model for bubble formation and weeping in quiescent liquid (Zhang & Tan, 2000). The situation analyzed is schematically shown in Fig. 1. The model predicts bubble volume, formation time and weeping rates are compared with model predictions.

2.1. Chamber pressure equation

The differential equation of chamber pressure fluctuation is the same as that in quiescent liquid

\[ V_tP_c = \gamma(P_sQ - P_cQ). \]  

2.2. Orifice equation

Orifice equation is defined as follows:

\[ \frac{dV_o}{dt} = k_b(\Delta P)^{0.5} \]  

\[ \Delta P = P_c - P_b, \quad k_b = \pi r_o^2 \sqrt{2/\rho_G C_G}, \]  

where \( C_G = 1.5 + 2 f b/r_o \) (Miyahara & Takahashi, 1984), \( f \) is the fanning friction factor.

2.3. Bubble pressure analysis

Assuming potential flow theory, i.e., irrotational and inviscid flow, the generalized Bernoulli equation

\[ \frac{P_t(r, \theta) - P_m}{\rho_L} = \frac{\partial \phi}{\partial t} - \frac{1}{2} |u|^2 \]  

can be applied to determine the liquid pressure at position \((r, \theta)\). Following Tsuge et al. (1981), the velocity potential \( \phi \) around a spherical bubble which is expanding and rising upward in a uniform transverse flow can be expressed as the sum of the potential \( \phi_T \) due to the rising motion, the potential \( \phi_p \) associated with the expansion and contraction of the bubble, and the potential \( \phi_F \) due to the uniform liquid flow. By potential flow theory, \( \phi_T = (a^2 U/2r^2) \cos \theta \) and \( \phi_p = a^2 \dot{a}/r \). To account for the liquid cross-flow, we superimpose a velocity potential term representing uniform flow parallel to the orifice plate at infinity, \( \phi_F = r U \cos \theta \). Note that the angle \( \theta \) is measured from the vertical bubble axis.

Hence, the velocity potential \( \phi \) can be expressed as

\[ \phi = \phi_p + \phi_T + \phi_F = \frac{a^2 \ddot{a}}{r} + \frac{a^3 U}{2r^2} \cos \theta + r U \cos \theta \]  

and the absolute liquid velocity is

\[ |u| = \sqrt{\left(\frac{\partial \phi}{\partial r}\right)^2 + \left(\frac{\partial \phi}{\partial \theta}\right)^2}. \]  

The average liquid pressure at the bubble boundary can be calculated from the equation:

\[ \bar{P}_L \int_{-s}^s 2 \pi a \, dx = \int_0^{\theta'} (2 \pi a \sin \theta P_L(r, \theta)a)_{r=a} \, d\theta, \]  

where \( \theta' = \cos^{-1}(-s/a) \) for \( s < a \) and \( \theta' = \pi \) for \( s \geq a \).

The bubble pressure is equal to the sum of the average liquid pressure at the bubble boundary(\( \bar{P}_L \)) plus the surface tension force. By substitution in Eq. (4) and rearranging, the final expression for bubble pressure is

\[ P_b = \rho_L \left[ \frac{3}{2} a^2 + a \dot{a} - gs + \frac{1}{2} U_t^2 \right] - \frac{\rho_L U_t^2 a}{4(s + a)} \times \left(1 + \frac{s}{2} - \frac{3 s^3}{2 a^3}\right) + P_p + \frac{2 \sigma}{a} \]  

\[ (7a) \]
2.4. Force balance for the bubble

The forces acting on the bubble in vertical and horizontal directions have been modeled separately. In the vertical direction, buoyancy force is dominant,

$$P_b = \rho_L \left( \frac{3}{2} \Delta a^2 + a\ddot{a} - g s + \frac{1}{2} U_i^2 \right) = \frac{\rho_L U^2}{4} + P_\infty + \frac{2\sigma}{a}. \quad (7b)$$

from which vertical rising velocity $U$ and the vertical position $s$ can be calculated.

Cross-flow liquid will impose an additional drag force on the bubble in the horizontal direction. For simplicity, the bubble is assumed to flow with the liquid at the same velocity. Although this assumption is more reasonable for higher liquid flow velocities, it is nevertheless shown from experimental images by Tsuge et al. (1981) that the mean horizontal velocity of the bubble centroid was approximately equal to the horizontal liquid velocity even for the relatively low liquid cross-flow velocity of 0.12 m/s.

2.5. Bubble detachment criterion

Referring to Fig. 1, the tilted angle of the bubble axis is determined by

$$\sigma = \tan^{-1} \left( \frac{U/t}{s} \right), \quad (9)$$

where $t$ is the bubble formation time, $s$ is the vertical distance between bubble center and orifice plate (Tan et al., 2000). The distance from bubble center to orifice center can be expressed as

$$x = \sqrt{s^2 + (U/t)^2} = s/\cos \sigma. \quad (10)$$

Tsuge et al. (1981) experimentally determined the relationship between $x$ and $a$ for low liquid flow velocity

$$\frac{x - a}{d_o} = 1 - 0.02U_i. \quad (11)$$

Following this criterion, the bubble will detach if

$$x \geq a + (1 - 0.02U_i)d_o. \quad (12)$$

2.6. Weeping rate

After bubble detachment, there is a possibility of liquid weeping if the pressure at orifice is larger than the instantaneous chamber pressure. The pressure at the orifice during liquid cross-flow (in the absence of bubble formation) is

$$P_D = P_\infty. \quad (13)$$

The driving force for weeping,

$$\Delta P_D = P_\infty - P_{cv}. \quad (14)$$

Finally, weeping will occur while $\Delta P_D > \Delta P_a$, where

$$\Delta P_a = \frac{2\sigma}{r_o} - \frac{2\rho_L g s}{3} - \rho_L bg \quad (15)$$

accounts for the effects of surface tension and the weight of liquid.

To determine weeping rate at a submerged single orifice, we can apply the gas flow equation inversely for the liquid flow

$$\frac{dV_L}{dT} = k_w (\Delta P_D)^{0.5} \quad (16)$$

where $k_w = \pi r_o^2 \sqrt{2/\rho_L C_L}$. $C_L$ is the orifice coefficient, is taken as 2.94 for an axially symmetrical orifice (McCann & Prince, 1969).

Integrating over one weeping period (from $T_1$ to $T_2$),

$$V_L = \int_{T_1}^{T_2} k_w (\Delta P_D)^{0.5} \, dT \quad (17)$$

and the average liquid weeping flow rate over one cycle will be given by

$$\bar{v}_L = \frac{V_L}{T_2 - T_1} = \frac{V_L}{t_b + T_m + T_w}. \quad (18)$$

During weeping or waiting process, the chamber pressure will rise gradually as a result of continuous gas inflow. Once the chamber pressure attains $P_c(0) = P_\infty + 2\sigma/r_o + 0.5\rho_L U_i^2$, the next bubble cycle will be initiated and the whole process will be repeated.
3. Experimental

The experimental set-up for study of bubble formation and weeping at a submerged orifice with liquid cross-flow is shown diagrammatically in Fig. 2. A centrifugal pump (2) transfers liquid water from the main reservoir (1), of capacity 75l, through the rotameter (3) and horizontal acrylic pipe, to the main bubble column (4). A rectangular downcomer and weir arrangement was used to introduce uniform and unseparated liquid cross-flow. The width and length of the cross-flow channel were 10 and 16 cm, respectively, and the weir height was 10 cm. Liquid overflow across the weir was returned to the main reservoir via a large diameter acrylic pipe.

As a preliminary study, liquid flow velocities were measured at various points in the channel using a Streamflo flowmeter (Series 400, Nixon Instrumentation Ltd., England). Fig. 3a shows the measured liquid cross-flow velocity versus volumetric liquid flowrate for three different positions in line with the orifice: 4 cm upstream of the orifice, at the orifice, and 4 cm downstream of the orifice. Fig. 3b shows the measured liquid cross-flow velocity versus volumetric liquid flowrate.
flowrate at positions across the flow channel at the orifice plane: 4 cm to the right and left of the orifice, and at the orifice. All readings were taken very close to the orifice plate (≈5 mm).

The results clearly show that cross-flow velocity is proportional to the volumetric flowrate at a specified position. There is a decrease of flow velocity from upstream to downstream, and this may be due to a slight increase in liquid height from the downcomer end to the weir end. The cross-flow velocity across the channel is almost uniform, with only a small decrease in velocity from the center of the channel towards the side walls, as shown in Fig. 3b. In the present investigation, liquid cross-flow velocity was based on the reading at the orifice.

Pressure fluctuations in the gas chamber were measured by a dynamic pressure transducer (Model 106B50, PCB PIEZOTRONICS). The signal was sent via a signal conditioner and 12-bit analog digital converter (ADC) into a data-logging computer. Bubble frequencies could be determined simply by Fourier transform of the pressure-time series data. Bubble volume were evaluated from the ratio of gas flowrate to bubbling frequency. Weeping rates were
Fig. 6. (a,b). Relationship between bubble frequency and gas velocity with different cross-flow velocities. $V_c = 7250 \text{ cm}^3$, $b = 1.6 \text{ mm}$, system: air–water.

measured by collecting and timing the liquid flow into a measuring cylinder. High-speed video camera images were employed to visualize the bubble formation and weeping phenomena.

4. Results and discussion

Figs. 4 and 5 show a set of high speed video images at an orifice with diameter 6.4 mm for chamber volume 7250 cm$^3$ in quiescent liquid and cross-flow liquid $U_l = 19 \text{ cm/s}$, respectively. The gas flow rates was 2.35 l/min (at standard conditions) for both cases. The time interval between frames is 40 ms.

It is evident that, for equivalent gas flowrates, the bubble formation time cycle is significantly shorter in the case of liquid cross-flow compared with no cross-flow. Furthermore, the weeping which accompanies bubble formation in the case of quiescent liquid (Fig. 4) is noticeably absent in the case of liquid cross-flow (Fig. 5).

Figs. 6a and b show the effects of liquid cross-flow velocity on bubble frequency at orifice diameters 4.8 and 6.4 mm, respectively. It was observed that with increasing of gas hole velocity, bubble frequency increases in both quiescent and cross-flow liquid as expected; an increase in liquid cross-flow velocity also led to an increase in bubble frequency. Some experimental difficulty was encountered in measuring bubble frequencies at high gas flow rates and
especially for the higher liquid cross-flow velocity (19 cm/s), because the bubbles were formed almost continuously and had a jet-like appearance. In spite of this, our simple model (which assumes spherical bubble shapes) appears to predict the values and trends of bubble frequency reasonably well.

The prediction of orifice weeping rates under conditions of liquid cross-flow is an important and unique feature of our model. Figs. 7 and 8 show the comparison between experimental weeping rates and model prediction with liquid cross-flow velocity as a parameter for orifice diameters 4.8 and 6.4 mm respectively.

Fig. 7 shows the results of 4.8 mm orifice diameter. Weeping rates significantly decrease with the introduction of liquid cross-flow, as seen from the experimental data. Clearly, agreement between theoretical prediction and experimental measurements is rather good.

Fig. 8 shows the effects of liquid cross-flow velocity on weeping rates at orifice diameter 6.4 mm. For this larger orifice, the low liquid cross-flow velocity of 11 cm/s had little impact on weeping rates compared to the case of quiescent liquid. Our model predicts this result very well. At a higher liquid cross-flow velocity of 19 cm/s, weeping rates are considerable lower; for the highest velocity of 29 cm/s,
there is virtually no weeping. The model also follows these trends well.

The influence of liquid cross-flow on weeping rates can be understood by observing value of chamber pressure during a bubble formation cycle. Fig. 9 shows the simulated values of chamber pressure with different liquid cross-flow velocities for \( d_o = 6.4 \text{ mm} \), \( V_c = 7250 \text{ cm}^3 \) and \( Q = 30 \text{ cm}^3/\text{s} \). The dotted line represents the value of hydrostatic pressure at orifice. It is clear that liquid cross-flow decreases the amplitude of the minimum chamber pressure and bubble formation cycle time (thereby increasing bubble frequency). This causes a decrease in weeping rate since (a) the driving force for weeping, determined by Eq. (14) is reduced, and (b) the time available for weeping is also reduced.

5. Conclusions

A theoretical model to predict bubble frequency and weeping rate at a submerged orifice with liquid cross-flow is presented, and validated with experimental data. The significant influence of liquid cross-flow in reducing weeping will have important implications for the design, operation and control of sieve trays processes.

Notation

- \( a \) bubble radius, m
- \( b \) thickness of plate, m
- \( C_L \) orifice coefficient for liquid flow, dimensionless
- \( C_G \) orifice coefficient for gas flow, dimensionless
- \( d_o \) orifice diameter, m
- \( f \) fanning friction factor, dimensionless
- \( g \) acceleration due to gravity, m.s\(^{-2}\)
- \( P_a \) gas pressure at inlet to chamber, Pa
- \( P_b \) bubble pressure, Pa
- \( P_c \) chamber pressure, Pa
- \( P_{cw} \) chamber pressure during weeping and waiting, Pa
- \( P_{D} \) orifice pressure during weeping and waiting, Pa
- \( P_{L}(r, \theta) \) liquid pressure at coordinate \((r, \theta)\), Pa
- \( P_{st} \) static pressure at orifice, Pa
- \( P_{st} \) average liquid pressure at bubble boundary, Pa
- \( P_{st} \) hydrostatic pressure at coordinate \((r, \theta)\), Pa
- \( P_{st} \) static pressure at orifice, Pa
- \( P_{st} \) = \( P_a + \rho_L g (s + r \cos \theta) \)
- \( Q \) average gas injection rate to the chamber, m\(^3\).s\(^{-1}\)
- \( q \) gas flow rate into bubble, m\(^3\).s\(^{-1}\)
- \( r_o \) orifice radius, m
- \( s \) the perpendicular distance between bubble center and orifice, m
- \( t \) time, s
- \( t_B \) total bubble formation time, s
- \( T \) time during weeping, s
- \( T_1 \) beginning time for weeping, s
- \( T_2 \) ending time for weeping, s
- \( T_w \) total weeping time, s
- \( T_{wt} \) total waiting time, s
- \( U \) bubble vertical rising velocity, m/s
- \( U_f \) uniform liquid cross-flow velocity across orifice, m/s
- \( V_B \) bubble volume, m\(^3\)
- \( V_L \) weeping liquid volume, m\(^3\)
- \( V_c \) chamber volume, m\(^3\)
- \( x \) distance between bubble center and orifice center, m
**Greek letters**

\( \gamma \)  
adiabatic exponent, dimensionless

\( \sigma \)  
tilted angle relative to the vertical, dimensionless

\( \phi \)  
velocity of potential, \( \text{m}^2 \cdot \text{s}^{-1} \)

\( \rho_G \)  
gas density, \( \text{kg} \cdot \text{m}^{-3} \)

\( \rho_L \)  
liquid density, \( \text{kg} \cdot \text{m}^{-3} \)

\( \sigma \)  
surface tension, \( \text{N} \cdot \text{m}^{-1} \)

\( \theta \)  
angular coordinate position, dimensionless

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**References**


