Bubble formation at multiple oriTZZces—bubbling synchronicity and frequency

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Abstract

Bubble formation at two, three, four and six symmetrical oriTZZces has been investigated by high-speed photography and analysis of chamber pressure fluctuations. Regimes of synchronous, alternative and unsteady bubbling were clearly identified, and the effects of oriTZZce spacing and liquid depth on bubbling synchronicity and frequency were studied. Theoretical predictions of bubble frequency and average radius were found to be in good agreement with experimental data when the bubble regime was highly synchronous.

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1. Introduction

The dispersion of gas bubbles in liquids plays an important role in bringing about efficient mass and heat transfer between the two phases. Important devices include bubble columns and sieve plate columns, in which bubbles are generated by introducing a stream of gas through oriTZZces into the liquid phase. Investigations on bubble formation mainly concern bubble frequency, size, shape, the influence of wake pressure of preceding bubbles and liquid weeping accompanying bubble formation and detachment.

As a fundamental phenomenon, bubble formation at single oriTZZce has been widely studied, although it is not in wide use practically. Numerous theoretical models have been developed in order to predict bubble size, shape, frequency and rising velocity in single oriTZZce bubbling (Davidson & Schütler, 1960a; Tan & Harris, 1986; Terasaka & Tsuge, 1990; Loubière & Hébrard, 2002). On the other hand, some experimental studies of bubble formation from industrial perforated plates have been undertaken (Zuiderweg, 1982; Wijn, 1998).

Few studies have addressed the case of multiple oriTZZces as an extension of single oriTZZce bubble formation. Titomanlio, Rizzo, and Acierno (1976) found that the bubble size generated at single oriTZZce approximates that of simultaneous bubbles at two oriTZZces with double the gas chamber volume and double the gas flow rate. Miyahara, Matsuba, and Takahashi (1983) investigated the size of bubbles generated from a perforated plate experimentally. For single oriTZZce bubbling, gas chamber volume was observed to play a very important role for determining bubble volumes and frequencies. However, for the bubble formation at multiple oriTZZces, they found that the effect of this parameter weakens as the number of oriTZZces is increased, and disappears when there were more than 15 oriTZZces. Ruzicka, Drahos, Zahradnik, and Thomas (1999, 2000) investigated bubble formation at two oriTZZces and identified two types of bubbling modes by means of analysis of pressure fluctuations in the gas chamber. They extended their work to more oriTZZces (three to thirteen) and found more types of bubbling modes based on various plate configurations. McCann (1969) developed a bubble interaction model to predict bubble frequencies placed in a line of five oriTZZces with 2.3 cm spacing. The results showed that bubble frequency depends on the total chamber volume, but the model did not appear to be applicable to other oriTZZce configurations.

There is a relative lack of fundamental understanding to link a comprehensive body of knowledge on single oriTZZces to industrial multi-oriTZZce distributors. For example, it is fairly obvious that even for two-oriTZZce bubbling, bubble sizes formed when both oriTZZces are bubbling simultaneously would be different from the case when the oriTZZces are “out of phase”. The degree of complexity would rise rapidly for

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three and more orifices. Therefore, the prediction of bubble sizes (or equivalently, bubbling frequency) for multi-orifice bubbling becomes a more difficult task when compared with single orifice bubbling.

This paper presents a systematic study of bubbling synchronicity and frequency for bubble formation at multiple (two-six) orifices. In particular, we seek to clarify the different modes with respect to synchronous, alternate and unsteady bubbling. Operating parameters investigated are gas flowrate, orifice spacing and liquid height. It is hoped that the study would increase our understanding of the factors affecting synchronicity and frequency in multi-orifice bubbling. We also propose a simple mathematical model to predict bubbling frequency in synchronous multi-orifice bubbling. Experimental results under various conditions are compared with the model predictions.

2. Model description

The model is a simple extension of the single-orifice model for bubble formation developed by Zhang and Tan (2000). The schematic diagram of the physical system (taking an example of two orifices) is shown in Fig. 1. Bubbles are assumed to remain spherical during formation and deform into spherical-cap bubbles after detachment. A detached and rising bubble is assumed to exert a wake pressure on the subsequent bubble forming at the orifice. Two significant simplifications are made: firstly, that the bubbling at the orifices is synchronous (i.e. in phase); and secondly, the side-by-side interaction of adjacent bubbles is not included in the modeling.

The model assumes that the bubble grows from an initial hemisphere to the complete spherical bubble until detachment. This period is defined as formation time \( t_f \). After bubble detachment, the pressure in the gas chamber will accumulate due to the continuous input of gas until the next hemispherical bubble appears. The time between detachment of the former bubble and the growth of the next bubble is defined as waiting time \( t_w \). There is no weeping phenomenon occurred during the waiting period. Thus the bubble formation period equals to the sum of \( t_f \) and \( t_w \). Bubble formation frequency is the inverse number of the period. The frequency here corresponds to the number of generated bubbles per phase.

\[
f = \frac{1}{t_f + t_w}
\]  

Thus the value of mean bubble volume is given as follows:

\[
V_B = \frac{Q}{N_{or} f}
\]  

where \( N_{or} \) is the number of orifices and \( Q \) is the volumetric gas flow rate into the gas chamber. The average gas velocity through each orifice is obtained by dividing the gas flow rate by the total area of orifices.

\[
V_g = \frac{Q}{N_{or} A}
\]  

where \( A \) is the area of each identical orifice.

The pressure in the gas chamber, \( P_c \), plays a significant role in determining bubble size and frequency. Following Zhang and Tan (2000) we apply an energy balance on the chamber and assume adiabatic and reversible conditions to obtain

\[
\dot{V}_c \frac{dP_c}{dt} = \frac{1}{CR} P_c (Q - N_{or} q)
\]  

The term \( N_{or} q \) represents the synchronous transient gas flow through orifices and \( \gamma \) is the adiabatic exponent for the gas.

Gas flow through each orifice is determined by the following orifice equation

\[
\frac{dV_B}{dt} = k_b \sqrt{P_c - P_b}
\]  

where \( k_b = \frac{\pi r_0^2 \sqrt{2/g C_G}}{} \) and \( C_G = 1.5 + 2 f'' b/r_0 \) (Miyahara & Takahashi, 1984), \( f'' \) is the fanning friction factor.

Following Zhang and Tan (2000), the transient bubble pressure, \( P_b \), and the liquid pressure at the orifice, \( P_{or} \), can be calculated by applying potential flow analysis for the surrounding liquid. In general, this allows the numerical evaluation of \( V_B \) at each time step using Eqs. (4) and (5).

Bubbles detach from the orifice if \( P_{or} \geq P_c \), which includes a criterion for necking (Zhang & Tan, 2000). After detachment, the waiting period starts and the pressure in the chamber will accumulate due to the continuous input of gas but no outflow of gas from chamber. The chamber pressure during the waiting time is derived from Eq. (4) under the condition \( q = 0 \):

\[
\ln P_c = \frac{\gamma}{V_c} Q T + \ln P_{c,DET}
\]  

Zhang and Tan (2000) calculate a wake pressure at the orifice, \( P_{wor} \), based on the rising velocity of a spherical cap.
bubble. With the accumulation of chamber pressure during the waiting period, the next bubble cycle will be initiated once the instantaneous pressure difference between chamber and orifice can overcome the effect of surface tension and wake pressure, which enables us to calculate the waiting time \( t_w \) from Eq. (6).

Thus, with appropriate initial conditions, the entire bubble formation and waiting cycle \( (t_f + t_w) \) can be calculated theoretically.

3. Experimental set-up and procedures

Fig. 2 shows the schematic diagram of the experimental apparatus. It consisted of a large Plexiglas® cylinder as the liquid chamber (4), the orifice insert, and a Plexiglas® cylindrical gas chamber volume (3). The upper column, designed conveniently for visual and photographic observations, had dimensions 190 mm ID and height of 470 mm and was open to atmosphere at the top. The interchangeable orifice insert below the liquid chamber allowed various orifice plates to be investigated. The volume of the gas chamber could be varied from 260 to 970 ml by tilting it partially.

Chamber insert below the liquid chamber allowed various orifice plates to be investigated. The volume of the gas chamber could be varied from 260 to 970 ml by tilting it partially with water. Gas inlet (6) and pressure transducer ports (5) were located at the upper part of the gas chamber at an angle of 90° to each other.

Purified air \((\gamma = 1.4)\) from the compressed gas cylinder (1) was introduced into the gas chamber. Air gas flow rate was controlled by means of three gas flow meters (2) with various ranges. Tap water was introduced into the liquid chamber through a rubber hose and the temperature was 20°C \((\rho_L = 1000 \text{ kg m}^{-3}, \sigma = 0.07 \text{ N m}^{-1})\). The liquid height was 10–30 cm.

Orifice plates were made of 1 mm thick stainless steel. Orifice diameter used was 1.6 mm. For two-orifice plates, two holes were placed symmetrically to the center with spacing of 1, 2, 3 and 4 cm, respectively. For the other plates with more than two orifices, holes were arranged symmetrically around a central circle with diameter 4 cm.

Pressure fluctuations in the gas chamber were recorded by a dynamic pressure transducer (106B50, PCB PIEZOTRONICS) and the output signals were fed into a computer through a 12-bit ADC (analog digital converter, PICO). Bubble frequencies were determined by Fourier transform of the pressure–time series data. Visualization images were taken by a high-speed video camera (NIKON, LF-N60M), capable of 250 frames per second.

4. Results and discussion

4.1. Bubbling modes at two orifices

Experiments for the air–water system were carried out at the following conditions in a two-orifice bubbling set-up: \( V_c = 480 \text{ cm}^3, H = 30 \text{ cm}, d_o = 1.6 \text{ mm}, s = 1 \text{ cm} \) with gas flow rates varying from about 0.83 to 8 \( \text{cm}^3 \text{ s}^{-1} \). By analysis of chamber pressure fluctuation signals and visual images from a high-speed video camera, three distinct modes of bubbling were observed.

Synchronous bubbling: A region of steady, synchronous bubbling begins at very low gas flow rates, up to about 2.5 \( \text{cm}^3 \text{ s}^{-1} \). Bubbles are formed simultaneously through both orifices at a regular frequency, as shown in Fig. 3(a) for \( Q = 2.5 \text{ cm}^3 \text{ s}^{-1} \). The corresponding chamber pressure signal (Fig. 4(a)) consists of peaks with uniform frequency and amplitude. Each peak represents one episode of twin bubble detachment. The twin bubble formation frequency can be easily determined via Fast Fourier Transform (FFT) analysis of the pressure signals (Fig. 5(a)) which shows a very distinct peak at the predominant frequency. Under such conditions, we consider that virtually 100% of the pressure peaks correspond to synchronous bubbling.

Alternate bubbling: As the gas flow rate is increased to 4.2 \( \text{cm}^3 \text{ s}^{-1} \), the bubbling becomes less steady and the proportion of synchronous bubbling decreases. Periods of synchronous bubbling are interspersed with alternate bubbling, i.e. only one of two orifices is bubbling at a particular instant of time, as depicted in Fig. 3(b). This phenomenon can be seen quite clearly in Fig. 4(b), where alternate bubbling regions are represented by peaks with approximately half the amplitude of synchronous bubbling regions. FFT analysis (Fig. 5(b)) reveals a more spread out profile as compared with 100% synchronous flow. Furthermore, a twin peak is discernible, probably indicating that synchronous and alternate bubbling regimes are occurring at slightly different frequencies. By counting the peaks in Fig. 5(b), it is estimated that the proportion of synchronous bubbling is 72%.

Unsteady bubbling: With further increase in gas flow rate to \( Q = 8.3 \text{ cm}^3 \text{ s}^{-1} \), the visual pattern of bubbling becomes very chaotic, and characterized by rapid multiple bubble formation at one or both orifices (Fig. 3(c)). This is represented by occasions of low amplitude, high frequency
Fig. 3. High-speed photographic images of bubble formation. $V_c = 480 \text{ cm}^3$, $d_o = 1.6 \text{ mm}$, $H = 30 \text{ cm}$, $s = 1 \text{ cm}$ and (a) $Q = 2.5 \text{ cm}^3 \text{s}^{-1}$, (b) $Q = 4.2 \text{ cm}^3 \text{s}^{-1}$, (c) $Q = 8.3 \text{ cm}^3 \text{s}^{-1}$.

Fig. 4. Chamber pressure fluctuations: $V_c = 480 \text{ cm}^3$, $d_o = 1.6 \text{ mm}$, $H = 30 \text{ cm}$, $s = 1 \text{ cm}$, (a) $Q = 2.5 \text{ cm}^3 \text{s}^{-1}$, (b) $Q = 4.2 \text{ cm}^3 \text{s}^{-1}$, (c) $Q = 8.3 \text{ cm}^3 \text{s}^{-1}$. Total time=20 s.
Fig. 5. FFT analysis of chamber pressure fluctuations: $V_c = 480$ cm$^3$, $d_o = 1.6$ mm, $H = 30$ cm, $s = 1$ cm, (a) $Q = 2.5$ cm$^3$ s$^{-1}$, (b) $Q = 4.2$ cm$^3$ s$^{-1}$, (c) $Q = 8.3$ cm$^3$ s$^{-1}$.

Fig. 6. Percentage of synchronous bubbling versus gas flow rate: $V_c = 480$ cm$^3$, $H = 30$ cm, $d_o = 1.6$ mm, and (a) $s = 1$ cm; (b) $s = 4$ cm.

The proportion of synchronous bubbling for a small orifice spacing of 1 cm shows a markedly different pattern (Fig. 6(a)). Clearly, the initial synchronous region is smaller, with $S_1$ occurring at about $Q = 2.5$ cm$^3$ s$^{-1}$. Above this flow rate, the trend of percent synchronous bubbling is generally decreasing with increasing flow rate. At about $Q = 8.7$ cm$^3$ s$^{-1}$, the percentage of synchronous bubbling becomes too chaotic to yield a peak frequency. This point, labeled $F_1$, represents the limit of measurability of a regular bubble frequency, and is close to the point where unsteady bubbling predominates.

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The decrease in synchronous bubbling for the 1 cm orifice spacing compared with 4 cm spacing can be...
explained by the higher bubble-to-bubble interaction and likelihood of coalescence in the liquid phase for closely spaced orifices (Fig. 3(c), for $s = 1$ cm). Variations in liquid pressure due to coalescence and the turbulent wake behind rising bubbles can lead to asymmetrical effects at the orifices, and the onset of asynchronous bubbling. Zhang and Tan (2000) have demonstrated the significant impact of wake pressure on subsequent bubble formation and weeping at a single orifice.

The interesting phenomenon of distinct regimes of highly synchronous bubbling at $Q = 3.2, 4.7,$ and $7.2$ cm$^3$ s$^{-1}$, as seen in Fig. 6(a) could be due to strongly resonant fluctuations in the gas chamber. The dimensionless capacitance number $N_c$, defined as $N_c = 4g(\rho_L - \rho_G)V_c\gamma/N_0\pi d_o^2 \rho_G c^2$ is used to quantify the chamber volume influence in the multiple-orifice bubbling system (Tsuge & Hibino, 1983). The value of $N_c$ in this work is 10.1, lying between the range of intermediate condition and constant pressure condition, which were proposed by Tadaki and Maeda (1963). Another dimensionless factor, dimensionless gas flow rate number $N_w$, governs bubbling at each orifice and $N_w = B_o F^3 r$, where $B_o$ is Bond number ($B_o = \rho d_o^2 g/\sigma$), $F$ is Froude number ($F = V_c^2/d_o g$) (Tsuge & Hibino, 1983).

A calculation of the dimensionless factor ($N_c/N_w$) at these points yields integral values of 5, 3 and 2. This suggests that highly synchronous bubbling may be encouraged when the bubbling frequency coincides with a multiple factor of the natural resonating frequency of the gas chamber.

Ruzicka et al. (2000) reported observing instances of synchronous bubbling at very low ($< 2$ cm$^3$ s$^{-1}$) and relatively high gas flow rate ($> 10$ cm$^3$ s$^{-1}$), with asynchronous bubbling in between. For their system, values of $N_c/N_w$ at 1.7 and 13 cm$^3$ s$^{-1}$ were estimated to be 7.9 and 1.0, respectively. It is possible that their observation of synchronous bubbling at high gas flowrate was caused by episodes of resonating frequency.

Fig. 7 shows the measured bubbling frequencies for orifice spacing of 1, 2, 3 and 4 cm in a two-orifice system. $S_1$ to $S_4$ shows the limit of initial synchronous bubbling regions for each spacing. Within the purely synchronous region of all four cases, the frequencies are almost identical, implying that the orifice spacing has no effect on bubbling frequency in synchronous bubbling. One would expect bubble–bubble interaction to be more pronounced for a small orifice spacing (say, 1 cm) than for a larger spacing (4 cm). Bubble–bubble interactions give rise to liquid pressure variations which result in a greater tendency towards unsteady bubbling. The data supports this view, since the limit of synchronous bubbling ($S_1, S_2, \text{etc.}$) shows an increasing trend as orifice spacing increases.

### 4.2.2. Liquid depth

Fig. 8 compares the frequency and synchronicity of two-orifice bubbling with different liquid depths of 10 and 30 cm. The other system parameters are: $V_c = 560$ cm$^3$, $d_o = 1.6$ mm, $s = 4$ cm. For gas flow rates up to $Q = 8.3$ cm$^3$ s$^{-1}$, the two sets of data are largely identical, which confirms the observation reported by numerous other investigators that liquid depth has virtually no effect on bubbling frequency (apart from very shallow liquid depths equivalent to a few orifice diameters) (Davidson & Amick, 1956; LaNauze & Harris, 1974). However, the liquid depth can be seen to significantly affect the synchronicity of bubbling. For a 30 cm depth, the limit of 100% synchronous bubbling ($S_{30}$) and limit of frequency measurement ($F_{30}$) occur at $Q = 5.0$ and 8.3 cm$^3$ s$^{-1}$, respectively. In the case of 10 cm depth, both $S_{10}$ and $F_{10}$ are greatly increased, to about $Q = 20$ cm$^3$ s$^{-1}$.

These observations may be partly explained by the relatively larger number of rising bubbles above the orifices in the case of the higher liquid depth. As many as 20 to 25 bubbles formed a rising chain above the orifices for the 30 cm liquid depth at high gas flowrates, giving rise to significant pressure fluctuations in their wake. While these transient wake pressures may have little effect on the average bubbling frequency, they would profoundly affect the synchronicity. For the smaller liquid depth, the number of wake-causing

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**Fig. 7.** Bubbling frequency and synchronicity for different orifice spacing: $V_c = 480$ cm$^3$, $H = 30$ cm, $d_o = 1.6$ mm.

**Fig. 8.** Bubbling frequency and synchronicity for different liquid depths: $V_c = 560$ cm$^3$, $d_o = 1.6$ mm, $s = 4$ cm.
bubbles was much lower owing to the shorter rising time of detached bubbles.

### 4.3. Comparison between experiment results and modeling

Fig. 9 compares the experimental data of bubbling frequency with model predictions for the three-orifice bubbling system with different gas chamber volumes. The corresponding percent of synchronous bubbling for each experimental run is listed in Table 1. It can be seen that discontinuities in the frequency data occurred at $V_g = 1.1 \text{ m s}^{-1}$ for $V_c = 260 \text{ cm}^3$ and $V_g = 1.6 \text{ m s}^{-1}$ for $V_c = 560 \text{ cm}^3$. From the photographic and chamber pressure images, it was observed that numerous small amplitude, high frequency bubbling bursts occurred at these regions. Near these regions of anomalous high frequency bursts, and in the case of the largest chamber volume ($V_c = 970 \text{ cm}^3$), the model under-predicts the bubbling frequency. This is expected, as our simple model assumed synchronous bubble formation, and these regions showed significantly less than 100% synchronous bubbling (Table 1).

![Fig. 9. Comparison of average gas velocity vs. frequency between model predictions and experimental data with chamber volume as a parameter.](image)

![Fig. 10. Comparison of frequency between model predictions and experimental data with orifice number as a parameter.](image)

Fig. 10 compares the experimental data of bubbling frequency with model predictions for different number of orifices, $N_{or}$. It can be seen that for the same average gas velocity through each orifice, the bubbling frequency increases with increasing $N_{or}$. The corresponding percent synchronous bubbling at each data point is shown in Table 2.

It is clear that the theoretical model is able to predict the experimental data and trends rather well, especially in regions of highly synchronous bubbling. Fig. 11 compares the frequency value in Figs. 9 and 10 between calculations and measurements in highly synchronous bubbling regions (above 90%). It can be seen that all the fits are good, with almost all the points lying within a range of ±15%.

### 5. Conclusions

This study provides clear experimental evidence of synchronous, alternate and unsteady bubbling regimes at multiple orifices. The degree of synchronicity generally decreases at high gas flowrates due to the onset of unsteady
Table 2
Percentage of synchronous signals with oriTZZce number \((N_{or})\) as a parameter. \(V_c = 480\, \text{cm}^3, d_o = 1.6\, \text{mm}, H = 30\, \text{cm}, s = 4\, \text{cm}, b = 1\, \text{mm}, \text{system: air–water}\)

<table>
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<th>(V_g) (m s(^{-1}))</th>
<th>(%\text{Sym}, N_{or} = 2)</th>
<th>(%\text{Sym}, N_{or} = 3)</th>
<th>(%\text{Sym}, N_{or} = 4)</th>
<th>(%\text{Sym}, N_{or} = 6)</th>
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Fig. 11. Measured vs. Calculated values of frequency. \(d_o = 1.6\, \text{mm}, H = 30\, \text{cm}, s = 4\, \text{cm}, b = 1\, \text{mm}, \text{system: air–water}\).

\(\circ\) \(N_{or} = 3; V_c = 260\, \text{cm}^3\); \(\bullet\) \(N_{or} = 3; V_c = 560\, \text{cm}^3\); \(\blacklozenge\) \(N_{or} = 3; V_c = 970\, \text{cm}^3\); \(\blacklozenge\) \(V_c = 480\, \text{cm}^3, N_{or} = 2\); \(\blacktriangle\) \(V_c = 480\, \text{cm}^3, N_{or} = 3\); \(\blacktriangle\) \(V_c = 480\, \text{cm}^3, N_{or} = 4\); \(\bigcirc\) \(V_c = 480\, \text{cm}^3, N_{or} = 6\).

bubbling. Both the oriTZZce spacing and liquid depth can af- affect the bubbling synchronicity via liquid pressure effects due to bubble-to-bubble interaction, coalescence and the wake pressure of preceding bubbles. A modified theoretical model for predicting synchronous bubble frequency in multiple-oriTZZce bubble formation has been presented, and predicted values of frequency under a variety of operating conditions agreed within ±15% with experimental data in the highly synchronous bubbling regime. These results should provide a sound basis for further fundamental studies into bubble formation phenomena at multiple oriTZZces.

### Notation

- \(A\): oriTZZce area, m\(^2\)
- \(b\): thickness of plate, m
- \(B_o\): Bond number, dimensionless
- \(c\): sound velocity in the gas, m s\(^{-1}\)
- \(d_o\): oriTZZce diameter, m
- \(C_G\): oriTZZce coefficient for gas flow, dimensionless
- \(D\): diameter of gas chamber, m
- \(f\): bubble frequency, s\(^{-1}\)
- \(f'\): fanning friction factor, dimensionless
- \(F_r\): Froude number, dimensionless
- \(g\): acceleration due to gravity, m s\(^{-2}\)
- \(H\): liquid height, m
- \(N_c\): capacitance number, dimensionless
- \(N_{uw}\): gas flow rate number, dimensionless
- \(N_{or}\): number of oriTZZces, dimensionless
- \(P_b\): bubble pressure, Pa
- \(P_c\): chamber pressure, Pa
- \(P_{DET}\): chamber pressure at bubble detachment, Pa
- \(P_{or}\): liquid pressure at oriTZZce, Pa
- \(P_{wo}\): wake pressure at oriTZZce, Pa
- \(q\): average gas flow rate through each oriTZZce, m\(^3\) s\(^{-1}\)
- \(Q\): average gas injection rate to the chamber, m\(^3\) s\(^{-1}\)
- \(r_o\): oriTZZce radius, m
- \(s\): spacing, m
- \(t\): time, s
- \(t_f\): bubble formation time, s
- \(t_w\): waiting time, s
- \(T\): time during waiting, s
- \(V_B\): bubble volume, m\(^3\)
- \(V_c\): chamber volume, m\(^3\)
- \(V_g\): average gas velocity through each oriTZZce, m s\(^{-1}\)

### Greek letters

- \(\gamma\): adiabatic exponent, dimensionless
- \(\rho_G\): gas density, kg m\(^{-3}\)
\[ \rho_l \quad \text{liquid density, kg m}^{-3} \]
\[ \sigma \quad \text{surface tension, N m}^{-1} \]

References


